

Mathematics 1



Université Libanaise

Chapter 1: Logic and Sets

Outline

1. Notions of Logic.

- Connectors of propositions and compound propositions
- Predicates and quantifiers.

2. Sets.

Chapter 1: Logic and Sets

1. Logic

1.1. Proposition.

- **Activity.**

- Answer True or False

- "4<1" ??.
- "6 is even" ??.
- "what time is it" ??.

(F T

- **Definition 1.** A proposition (or assertion) is a statement that is either true or false. It cannot be true and false.

" 0 ≠
" 0

Chapter 1: Logic and Sets

1. Logic

1.1. Proposition.

- **Examples.**

- "4<1" is a false proposition.
- "6 is even" is a true proposition.
- "what time is it" is not a proposition.

- **Truth Values.**

- The truth value of a proposition is:
- Let it be true, in this case we match T,
- Either false, in this case, we match F.
- A truth table is a table grouping together propositions with their truth values.

Chapter 1: Logic and Sets

1. Logic

1.1. Proposition.

Handwritten notes in red ink:

$$\neg P \quad P: "x \leq 6" \quad T \quad P: "x \neq 0" \quad T$$

$$\neg P: "x > 6" \quad F \quad \neg P: "x = 0" \quad F$$

- **Definition 2.** The negation of a proposition P , is the proposition "it is not true that we have P ", denoted by $\bar{P}$ or *not* P or $\neg P$, which is such that:

$\bar{P}$ is true when P is false,
and, $\bar{P}$ is false when P is true.

Thus, the truth table of $\bar{P}$ is:

P	$\bar{P}$
T	F
F	T

P	$\bar{P}$
T	F
F	T

Chapter 1: Logic and Sets

1. Logic

1.1. Proposition.

• Examples

- Let P : " -2 is negative" (T)
- $\bar{P}$: "It is false that -2 is negative" (F).
- Let P : " $1 + 5 = 6$ " (T)
- $\bar{P}$: " $1 + 5 \neq 6$ " (F).
- Let P : " $2 > 6$ " (F)
- $\bar{P}$: " $2 \leq 6$ " (T).
- Let Q : " $-2 = 6$ " (F)
- $\bar{Q}$: " $-2 \neq 6$ " (T).



$\bar{P}$: " $-$ is positive"

P : " $3 \leq 1$ " F

$\bar{P}$: " $3 > 1$ " F

Chapter 1: Logic and Sets

1. Logic

1.1. Proposition.

- There are two kinds of propositions:
 - Simple propositions.
 - Compound propositions which are formed from several simple propositions.
- **Example.**
 - Let P : " $2 + 3 = 11$ " and Q : " $5 + 1 = 15$ ".
 - The proposition P is simple, as well as the proposition Q which is simple.
 - Consider the proposition R : [$"2 + 3 = 11"$ and " $5 + 1 = 15$ "],
 - R is a compound proposition, formed from two simple propositions and a connector.
 - In the following part, we will introduce the proposition connectors and we will study the truth value of the composed propositions following the use of each connector.

$R: ["2 < 6" \text{ and } "1 > 100"]$

Chapter 1: Logic and Sets

1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

In the following, P and Q are two propositions.

a. The Conjunction “and”

- **Activity.** Give the truth value for each of the following propositions.

$$\left[\underset{\text{T}}{\text{"2+3=5"}} \wedge \underset{\text{T}}{\text{"5 + 6 = 11"}} \right] \text{ : } \text{T}$$

$$\left[\underset{\text{T}}{\text{"2+3=5"}} \wedge \underset{\text{F}}{\text{"5 + 6 = 15"}} \right] \text{ : } \text{F}$$

$$\left[\overset{\text{F}}{\text{"2+3=7"}} \wedge \overset{\text{T}}{\text{"5 + 6 = 11"}} \right] \text{ : } \text{F}$$

$$\left[\overset{\text{F}}{\text{"2+3=7"}} \wedge \underset{\text{F}}{\text{"5 + 6 = 15"}} \right] \text{ : } \text{F}$$

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1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

a. The conjunction “and”

- **Definition 3:** We call the conjunction of P and Q , the proposition " P and Q ", denoted by " $P \wedge Q$ ".

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table

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1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

b. The disjunction “or”

• **Activity.** Give the truth value for each of the following propositions.

• $"2+3=5" \vee "5 + 6 = 11"$

• $"2+3=5" \vee "5 + 6 = 15"$

• $"2+3=7" \vee "5 + 6 = 11"$

• $"2+3=7" \vee "5 + 6 = 15"$

[T T] T
[F T] T

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1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

b. The disjunction “or”

- **Definition 4:** We call the disjunction of P and Q , the proposition " P or Q ", denoted by " $P \vee Q$ ".

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Table

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1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

c. Implication " $\Rightarrow$ "

Activity. Give the truth value for each of the following propositions.

• $[(1 \underset{T}{\leq} 2) \Rightarrow (2 \underset{T}{\leq} 3)] : T$

$$1 \not\leq 2 + 1$$

• $[(1 \overset{F}{\leq} 2) \Rightarrow (2 \overset{F}{>} 3)] : F$

$$2 > 3$$

• $[(0 \overset{F}{=} 2) \Rightarrow (2 \overset{T}{=} 2)] : T$

• $[(1 \overset{F}{>} 2) \Rightarrow (2 \overset{F}{>} 3)] : T$

eg $(1 < 2) \Rightarrow (2 < 3)$
Then

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1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

c. Implication " $\Rightarrow$ "

- **Definition 5:** We define the proposition, denoted by " $P \Rightarrow Q$ ", which can be read: " **P implies Q** " and which also reads: 
- "**If P , then Q** " by the proposition " $not P$ or Q " (denoted by $\bar{P} \vee Q$).

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table

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1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

d. Equivalence " $\Leftrightarrow$ "

- We define a new proposition noted " $P \Leftrightarrow Q$ ", which can be read " P is **equivalent** to Q " or *P if, and only if, Q* , by the proposition:
- $[(P \Rightarrow Q) \wedge (Q \Rightarrow P)]$.

$P \Leftrightarrow Q$

P	Q	$\bar{P}$	$\bar{P} \vee Q$	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$
T	T	F	T	T	T	T
T	F	F	F	F	T	F
F	T	T	T	T	F	F
F	F	T	T	T	T	T

Truth Table

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1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

d. Equivalence " $\Leftrightarrow$ "

- **Remark.** $P \Leftrightarrow Q$ is true in cases where the truth values of P and Q coincide. We will say subsequently that two propositions are equivalent if they have the same truth table.
- **Example:** (" $1 < 0$ " $\Leftrightarrow$ "6 is even") is ^Ffalse.
_F _T
- **Remark.**
- A tautology is a proposition that is always true.
- A contradiction is a proposition that is always wrong.

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1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

- **Property.**

- $\overline{\overline{P}} \Leftrightarrow P$
- $P \vee \overline{P}$ is a tautology.
- $P \wedge P \Leftrightarrow P$
- $P \vee P \Leftrightarrow P$
- $$\left. \begin{array}{l} \overline{(P \vee Q)} \Leftrightarrow (\overline{P} \wedge \overline{Q}) \\ \overline{(P \wedge Q)} \Leftrightarrow (\overline{P} \vee \overline{Q}) \end{array} \right\} \text{De Morgan Rules}$$
- $$\left. \begin{array}{l} [P \wedge (Q \vee R)] \Leftrightarrow [(P \wedge Q) \vee (P \wedge R)] \\ [P \vee (Q \wedge R)] \Leftrightarrow [(P \vee Q) \wedge (P \vee R)] \end{array} \right\} \text{distribution rule of " } \wedge \text{ " over " } \vee \text{ " and of " } \vee \text{ " over " } \wedge \text{ " .}$$
- $\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$
- $(P \Rightarrow Q) \Leftrightarrow (\overline{Q} \Rightarrow \overline{P})$
- Show that $P \wedge \overline{P}$ is a contradiction

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1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

- **Property.**

$\neg(\neg P) \equiv P$

- $\overline{\overline{P}} \Leftrightarrow P$

P	$\overline{P}$	$\overline{\overline{P}}$
T	F	T
F	T	F

- They are equivalent because they have the same truth table.

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1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

- **Property.**

- $P \vee \bar{P}$ is a tautology

P	$\bar{P}$	$P \vee \bar{P}$
T	F	T
F	T	T

- Thus, this is a Tautology

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1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

- **Property.**

- $P \wedge P \iff P$

P	P	$P \wedge P$
T	T	T
F	F	F

- $(P \wedge P)$ and P are equivalent because they have the same truth table

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1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

- **Property.**

- $P \vee P \Leftrightarrow P$

P	P	$P \vee P$
T	T	T
F	F	F

They are $(\Rightarrow)$

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1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

- **Property.**

- $\overline{(P \vee Q)} \Leftrightarrow (\bar{P} \wedge \bar{Q})$

P	Q	$P \vee Q$	$\overline{P \vee Q}$	$\bar{P}$	$\bar{Q}$	$\bar{P} \wedge \bar{Q}$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

- $\overline{(P \vee Q)} \Leftrightarrow (\bar{P} \wedge \bar{Q})$ Because they have the same truth table.

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1. Logic

1.1. Proposition.

1.1.1. Connectors of propositions and compound propositions

- **Property.**

- $\overline{(P \wedge Q)} \Leftrightarrow (\bar{P} \vee \bar{Q})$

P	Q	$P \wedge Q$	$\overline{P \wedge Q}$	$\bar{P}$	$\bar{Q}$	$\bar{P} \vee \bar{Q}$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

- $\overline{(P \wedge Q)} \Leftrightarrow (\bar{P} \vee \bar{Q})$ Because they have the same truth table.

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- **Property.**

- $[P \wedge (Q \vee R)] \Leftrightarrow [(P \wedge Q) \vee (P \wedge R)]$

$$a(b+c) = (a \times b) + (a \times c)$$

P	Q	R	$Q \vee R$	$[P \wedge (Q \vee R)]$	$P \wedge Q$	$P \wedge R$	$[(P \wedge Q) \vee (P \wedge R)]$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

- They have the same truth table

Chapter 1: Logic and Sets

- **Property.**

- $[P \vee (Q \wedge R)] \Leftrightarrow [(P \vee Q) \wedge (P \vee R)]$

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$P \vee Q$	$P \vee R$	$[(P \vee Q) \wedge (P \vee R)]$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

- $[P \vee (Q \wedge R)] \Leftrightarrow [(P \vee Q) \wedge (P \vee R)]$ Because they are equivalent.

Chapter 1: Logic and Sets

- **Property.**

- $[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$

P	Q	R	$(P \Rightarrow Q)$	$(Q \Rightarrow R)$	$[(P \Rightarrow Q) \wedge (Q \Rightarrow R)]$	$P \Rightarrow R$	$[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Thus, this is a Tautology

Chapter 1: Logic and Sets

2. Predicates and Quantifiers.

2.1. Predicates

- **Activities.**

- Answer True or False

- 1) " $x > 1$ "
- 2) " $x^2 = 1$ "

- **Definition 7.** We call predicate, a proposition P whose truth values depend on one or more variables.

Chapter 1: Logic and Sets

2. Predicates and Quantifiers

2.1. Predicates .

- **Examples.**

- Answer True or False
 - 1) " $x > 1$ "
 - 2) " $x^2 = 1$ "
- The truth values depend on the value of x .

Chapter 1: Logic and Sets

2. Predicates and Quantifiers

2.2. Quantifiers



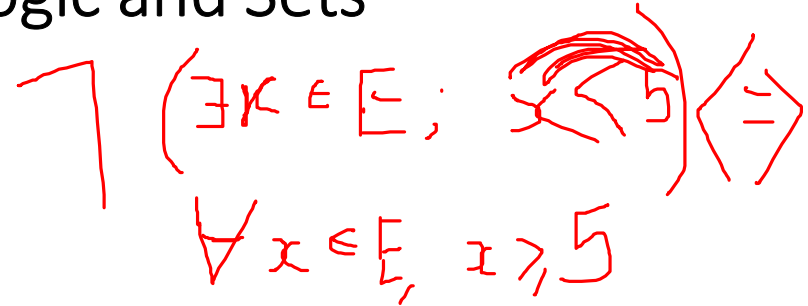
- **Activities.**

- The predicate "For all $x \in E$, we have x is greater than 2" can be written " $\forall x \in E, x > 2$ ".
- The predicate: "There exists at least one x of E , such that $x > 2$ " can be written " $\exists x \in E, x > 2$ ".
- **Definition 8.** Symbols " $\exists$ " and " $\forall$ " are called **quantifiers**.
- The existential quantifier $\exists$ means "there is at least one".
- The universal quantifier $\forall$, means "for all".
- The quantifier " $\exists!$ " means "there is one" or "there is one and only one".

Chapter 1: Logic and Sets

2. Predicates and Quantifiers

2.2. Quantifiers


$$\neg (\exists x \in E; x < 5) \Leftrightarrow \forall x \in E, x \geq 5$$

- **Examples.**

- Set the appropriate quantifier so that each of the Predicates is true:

- $\exists! x \in \mathbb{Q}$ such as $5x + 1 = 0$.

- $\forall x \in \mathbb{N}$ such as $x \geq 0$. $\mathbb{N} = \{0, 1, 2, 3, \dots\}$

- **Properties.** (negation of predicates with quantifiers)

- Let E be a set and $\wp$ a property defined on E . We have :

- $\neg [\forall x \in E, \wp(x)] \Leftrightarrow [\exists x \in E, \neg \wp(x)]$

- $\neg [\exists x \in E, \wp(x)] \Leftrightarrow [\forall x \in E, \neg \wp(x)]$

Chapter 1: Logic and Sets

2. Predicates and Quantifiers

2.2. Quantifiers

- **Remark.** In practice, to give the negation of a predicate containing one or more quantifiers, it is necessary to rewrite the predicate by replacing all the " $\forall$ " by " $\exists$ " and vice versa, respecting the order of the quantifiers, then by applying the negation to the the given property.

- **Examples.**

- $\neg(\forall x \in E, x > 2) \Leftrightarrow$

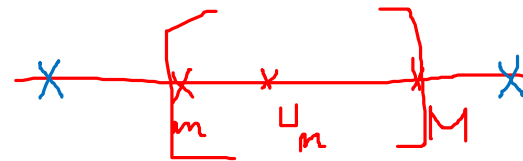
- $(\exists x \in E, x \leq 2)$

- $\neg(\exists x \in E, x > 2) \Leftrightarrow$

- $(\forall x \in E, x \leq 2)$

- $\neg(\exists m \in \mathbb{N}, \exists M \in \mathbb{N} \text{ such as } \forall n \in \mathbb{N}, m \leq u_n \leq M) \Leftrightarrow$

- $(\forall m \in \mathbb{N}, \forall M \in \mathbb{N}, \exists n \in \mathbb{N} \text{ such as } u_n < m \text{ ou } u_n > M)$



Exercise 2: (4 points)

- a. Say if the following proposition is a tautology (always True):

$$(\neg p \vee q) \vee (\neg q \vee p).$$

- b. Let p be the proposition "Antoine studies" and q the proposition "Antoine will have more than 20 out of 30 on the Math 1". Translate the following sentences into propositional language by replacing "*Antoine studies*" with p and "*Antoine will have more than 20 out of 30 on Math 1*" by q .

- b.1. If Antoine studies, then Antoine will have more than 20 out of 30 on Math 1.

$$p \Rightarrow q$$

- b.2. Antoine does not study.

$$\overline{p}$$

- b.3. Antoine will don't have more than 20 out of 30 on Math 1 or Antoine studies.

$$\overline{q} \vee p$$

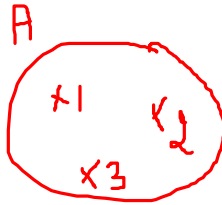
- b.4. Antoine does not study and Antoine will not have more than 20 out of 30 on the Math 1.

$$\overline{p} \wedge \overline{q}$$

Chapter 1: Logic and Sets

3. Sets.

3.1. Introduction.



- **Activity.**

- Let $A = \{1; 2; 3\}$. A is a set and 1, 2 and 3 are elements of A .

- **Definition 9.** A set E is a collection of objects. Objects are called elements.

- We note: $x \in E$ if an element x belongs to the set E $2 \in A$

- And if the item does not belong to the set. We note $x \notin E$. $5 \notin A$

- Example: let $A = \{1; 2; 3\}$. In this case, $2 \in A$ but $4 \notin A$.

- **Remark.**

- *The empty set is the set that has no elements. We denote it $\emptyset$. $\phi = \{\}$.*

- *A singleton is a set with a single element. Example: $B = \{a\}$.*

Chapter 1: Logic and Sets

3. Sets.

3.1. Introduction.

- **A set can be defined in two ways:**

- By listing all of its elements. We say that the set, in this case, is defined by extension. The order in which the elements are given does not play a role in the definition of the whole. So, $\{1, 2, 3\} = \{2, 1, 3\}$.
- By writing a property ϕ verified by all its elements. We say that the set is defined by intension. Then, $E = \{x \text{ such that } \phi(x)\}$.

- **Examples.**

- The set of even natural integers can be defined by intension as follows: $E = \{n \in \mathbb{N} / \exists p \in \mathbb{N} \text{ such that } n = 2p\} = \{0, 2, 4, \dots\}$ **In extension.**
- ~~• The set $F = \{0; 1\}$ is defined by extension, it is also defined by intension as follows: $F = \{n \in \mathbb{N}; n^2 = n\}$.~~

Chapter 1: Logic and Sets

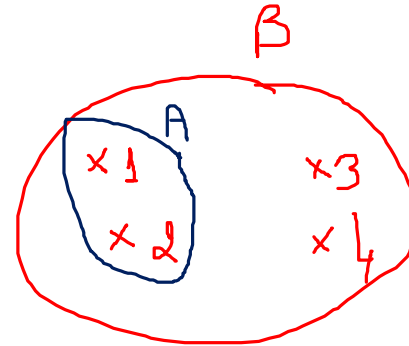
3. Sets.

3.2. Sets Operations.

3.2.1. Subsets.

- **Activity.**

- Let $A = \{1; 2\}$ and $B = \{1; 2; 3; 4\}$.
- All the elements of A belong to B .
- We have: $A \subset B$.



$E \not\subset$
 $\subset \not\subset$

- **Definition 10.** Let A and B be two Sets. We say that A is a subset of B , if every element of A is an element of B . We note: $A \subset B$.
- We say that A is a subset of B , or also that A is a part of B .
- **Example.** Let $A = \{a; b, c\}$ and $B = \{a, b, 3, 5, c\}$. We have: $A \subset B$.
- Let $A = \{1, 2\}$ and $B = \{1, 3, 4, 5\}$. We have: $A \not\subset B$.
- **Property.** We always have for any set E ($\forall E$): $\emptyset \subset E$ and $E \subset E$
2 $\notin B$ Thus, $A \not\subset B$

Chapter 1: Logic and Sets

3. Sets.

3.2. Sets Operations

3.2.1. Subsets.

- **Remarks.**

- Do not confuse between a singleton $\{x\}$ and an element x .
- Moreover, Let $A = \{1, 2, 3\}$, we have, $2 \in A$ but $\{2\} \subset A$.
- For the set $\{0; \{0\}\}$, use $\in$ or $\subset$
- $0 \notin \{0; \{0\}\}$; note that $0 \neq \{0\}$
- $\{0\} \in \{0; \{0\}\}$, and $\{0\} \subset \{0; \{0\}\}$
- $\{\{0\}\} \subset \{0; \{0\}\}$ note that $\{\{0\}\} \notin \{0; \{0\}\}$

$$A = \{1\}$$

$$B = \{1, 2, 3\}$$

$$\begin{array}{l} 1 \in B \\ \neq \\ \{1\} \subset B \end{array}$$

Chapter 1: Logic and Sets

3. Sets.

3.2. Sets Operations.

3.2.2. Power set.

- **Activity.**

Let $E = \{1,2\}$. We have, $\wp(E) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$.

- **Definition 11.** Let E be a set. All the subsets of E constitute a set called the power set of E . It is noted: $\wp(E)$.
- We say that: $A \subset E \Leftrightarrow A \in \wp(E)$; each subset of E is an element of $\wp(E)$;
- **Example** $\triangleright A = \{1\}$, we have $A = \{1\} \subset E$ then $A = \{1\} \in \wp(E)$.
- We call cardinal of a set E , denoted by $Card(E)$, the number of elements of E
- For any set E , we have: $Card(\wp(E)) = 2^{card(E)}$. **Here,** $Card(\wp(E)) = 2^2 = 4$.

Chapter 1: Logic and Sets

3. Sets.

3.2. Sets Operations.

3.2.2. Power set.

• **Example.** Let $E = \{1; 2; 3\}$, we have:

• $\wp(E) = \{\emptyset; \{1\}; \{2\}; \{3\}; \{1; 2\}; \{1; 3\}; \{2; 3\}; \{1; 2; 3\}\}.$

• **Remark.**

• $\wp(E) \neq \emptyset.$ $\emptyset = \{\}$

• $\wp(\emptyset) = \{\emptyset\}.$

• $\wp(\{\emptyset\}) = \{\emptyset; \{\emptyset\}\}.$

$E = \{\emptyset\},$

$\wp(\emptyset) = \{\emptyset\}$

$\wp(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$

$|\wp(\emptyset)| = 2^{|\emptyset|} = 2^0 = 1$

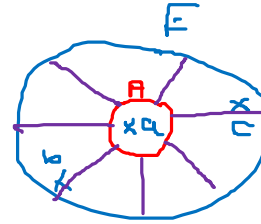
$|\wp(\{\emptyset\})| = 2^{|\{\emptyset\}|} = 2^1 = 2$

Chapter 1: Logic and Sets

3. Sets.

3.2. Sets Operations.

3.2.3. Complement of a set.



- **Activity.** Let $E = \{a, b, c\}$ and $A = \{a\}$.
- We have, $A \subset E$, then C_E^A exists and $C_E^A = \{b, c\}$.
- **Definitions 12.**
- Let E be a set and A, B two parts of E .
- Complementary to a set: we call complementary to A in E , the set of elements of E which do not belong to A . We note it: C_E^A or $\bar{A}^E$. Then,
- $C_E^A = \{x \in E / x \notin A\} = \bar{A}^E$
- Note: For C_E^A to be defined, it is necessary that $A \subset E$. This is a condition that must be verified in order for this set to exist.

Chapter 1: Logic and Sets

3. Sets.

3.2. Sets Operations.

3.2.3. Complement of a set.

- **Examples.**

- $C_{\mathbb{R}}^{\{0\}} = \mathbb{R}^*$.
- Let $E = \{1; 2; 3; 4\}$ and $A = \{1; 3\}$. Since $A \subset E$, then C_E^A exists and $C_E^A = \{2; 4\}$.
- Let $E = \{1; 2; 3; 4\}$ and $A = \{5; 2; 1\}$. In this case, C_E^A doesn't exist because $A \not\subset E$.

- **Properties.**

- For all $x \in E$, we have,
- $(x \in C_E^A) \iff (x \notin A)$
- $(x \notin C_E^A) \iff (x \in A)$

- $C_E^E = \emptyset$

- $C_E^\emptyset = E$

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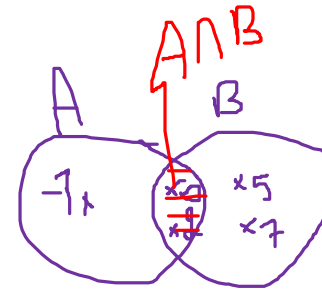
3. Sets.

3.2. Sets Operations.

3.2.4. Intersection of Sets.

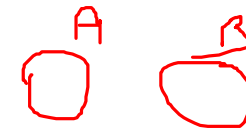
- **Activity.**

- Let $A = \{-1; 2; 0\}$ and $B = \{5; 7; 2; 0\}$.
- We have $A \cap B = \{0; 2\}$.



- **Definitions 13.**

- We call the intersection of two Sets A and B , which we denote by $A \cap B$, the set of elements of E belonging to both A and B . then,
- $A \cap B = \{x \in E / (x \in A) \wedge (x \in B)\}$.



- **Remark.** we say that A and B are disjoint when $A \cap B = \emptyset$. In other words, when A and B have nothing in common.

Chapter 1: Logic and Sets

3. Sets.

3.2. Sets Operations sur les Sets.

3.2.4. Intersection of Sets.

- **Examples.**

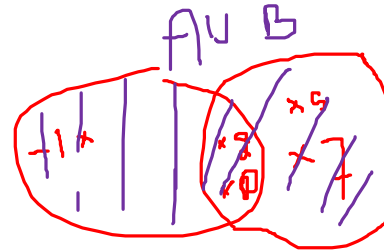
- Let $A = \{a, b, c, \{3\}\}$ and $B = \{a, e, f\}$. We have $A \cap B = \{a\}$.
- Let $A = \{-1; 1; 2\}$ and $B = \{3; 4\}$. Sets A and B are disjoint.

Chapter 1: Logic and Sets

3. Sets.

3.2. Sets Operations.

3.2.5. Union of Sets.



- **Activity.**

- Let $A = \{-1; 2; 0\}$ and $B = \{5; 7; 2; 0\}$. Find $A \cup B$.

- We have, $A \cup B = \{-1; 0; 2; 5, 7\}$.

- **Definition 14.**

- We call the union of two Sets A and B , which we denote by $A \cup B$, the set of elements of E belonging to A or B . then,

- $A \cup B = \{x \in E / (x \in A) \vee (x \in B)\}$.

- **Example.** Let $A = \{1; 2; 3\}$ and $B = \{1; -1; 2\}$.

- In this case, $A \cup B = \{-1; 1; 2; 3\}$.

Chapter 1: Logic and Sets

3. Sets.

3.2. Sets Operations.

3.2.6. Differences of Sets.

- **Activity.**

- Let $A = \{-1; 2; 0\}$ and $B = \{5; 7; 2; 0\}$.
- Find $A - B$.
- We have, $A - B = \{-1\}$.

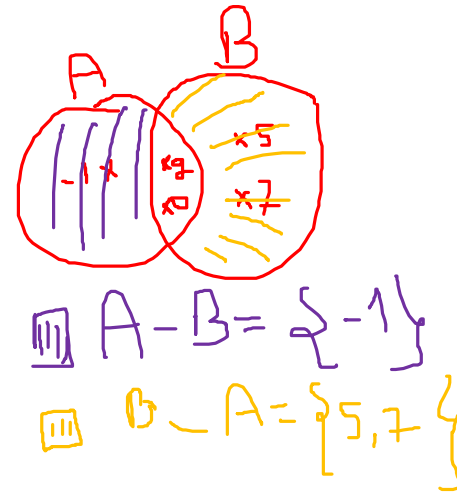
- **Definitions 15.**

- We call the **différence de A et de B**, denoted by $A - B$ (or $A \setminus B$) the set of elements of E that belong to A , but do not belong to B .

- $A - B = \{x \in E / (x \in A) \wedge (x \notin B)\}$.

- **Examples.** Find $B - A$.

- $B - A = \{5, 7\}$
- $\mathbb{N} \setminus \{0\} = \mathbb{N}^*$



Chapter 1: Logic and Sets

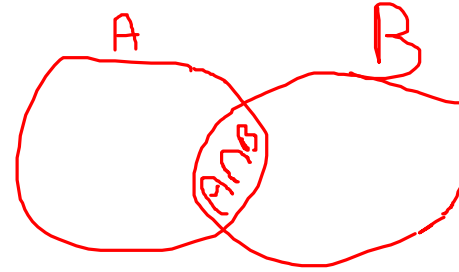
3. Sets.

3.2. Sets Operations.

3.2.6. Differences of Sets.

- **Properties.**

- $\cup$ is associative and commutative, the same for $\cap$.
- $\cap$ is distributive compared to $\cup$ and vice versa:
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
- $A \subset A \cup B$ and $B \subset A \cup B$.
- $A \cap B \subset A$, $A \cap B \subset B$ and $A \cap B \subset A \cup B$.
- $\emptyset \cap A = \emptyset$ and $\emptyset \cup A = A$.



Chapter 1: Logic and Sets

3. Sets.

3.2. Sets Operations.

3.2.6. Partitions.

• Activity.

- Let $E = \{1,2,3\}$.

- We have $A_1 = \{1\}; A_2 = \{2\}$ et $A_3 = \{3\}$ a partition of E .

• Definition 15.

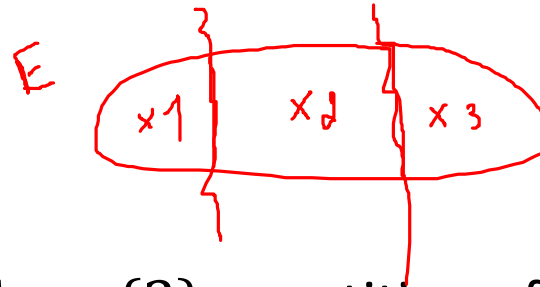
- Let E be a set. A family of subsets of $E: A_1, A_2, \dots, A_n$ is a **partition** of E , if the following conditions are verified:

- $\forall i, A_i \neq \emptyset$ ✓

- $\forall i, j$ such as $i \neq j, A_i \cap A_j = \emptyset$ ✓

- $\bigcup_{i=1}^n A_i = A_1 \cup A_2 \cup \dots \cup A_n = E$. ✓

- **Example.** Let $E = \{1,2,3\}$. Find the partitions of E .



2. $A_1 = \{1,2\}, A_2 = \{3\}$
 $A_1 \neq \emptyset, A_2 \neq \emptyset$
 $A_1 \cap A_2 = \emptyset, A_1 \cup A_2 = E$

3. $A_1 = \{1,3\}, A_2 = \{2\}$
4. $A_1 = \{2,3\}, A_2 = \{1\}$

Exercise 3. Answer the following questions :

1. Does the notations ϕ , $\{\phi\}$ and $\{\{\phi\}\}$ describe the same set ?
2. We consider the following subsets of $\mathbb{N}$: $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{1, 3, 5, 7\}$, $C = \{2, 4, 6\}$, $D = \{3, 6\}$.
 - (a) Determine $B \cap D, C \cap D$.
 - (b) Determine $B \cup C, C \cup D$
 - (c) Determine the complement in A of B , C and D .

Solution.

1. They are different sets. Well Studied.

2.1. We have, $B \cap D = \{3\}; C \cap D = \{6\}$

2.2. We have, $B \cup C = \{1, 2, 3, 4, 5, 6, 7\}; C \cup D = \{2, 3, 4, 6\}$

2.3. We have, $\bar{B}^A = \{2, 4, 6\}; \bar{C}^A = \{1, 3, 5, 7\}$ and $\bar{D}^A = \{1, 2, 4, 5, 7\}$.